

Matrix to Reduced Echelon Form, Step-by-Step

This handout accompanies [the YouTube video](https://www.youtube.com/watch?v=7piTfgtNa_U) - https://www.youtube.com/watch?v=7piTfgtNa_U

System of 3 equations

$$2x + 5y + 4z = 12$$

$$3x + y - 4z = 1$$

$$x - 2y - 3z = 0$$

Create the matrix.

$$\left| \begin{array}{cccc} 2 & 5 & 4 & 12 \\ 3 & 1 & -4 & 1 \\ 1 & -2 & -3 & 0 \end{array} \right|$$

The goal:

$$\left| \begin{array}{cccc} 1 & 0 & 0 & ? \\ 0 & 1 & 0 & ? \\ 0 & 0 & 1 & ? \end{array} \right|$$

Organize your work like this to show step-by-step how you put a matrix in reduced row echelon form.

Updated matrix

Plan for this step

Calculator step

$$\left| \begin{array}{cccc} 2 & 5 & 4 & 12 \\ 3 & 1 & -4 & 1 \\ 1 & -2 & -3 & 0 \end{array} \right|$$

$$R_1 \leftrightarrow R_3$$

MATRIX > MATH > C: rowSwap([A],1,3)

Verify output is correct

STO→[A]

$$\left| \begin{array}{cccc} 1 & -2 & -3 & 0 \\ 3 & 1 & -4 & 1 \\ 2 & 5 & 4 & 12 \end{array} \right|$$

$$-3(R_1) + R_2 \rightarrow R_2$$

$$\begin{array}{cccc} -3 & 6 & 9 & 0 \\ 3 & 1 & -4 & 1 \\ \hline 0 & 7 & 5 & 1 \end{array}$$

MATRIX > MATH > F:* row+(-3,[A],1,2)

Verify output is correct

STO→[A]

$$\left| \begin{array}{cccc} 1 & -2 & -3 & 0 \\ 0 & 7 & 5 & 1 \\ 2 & 5 & 4 & 12 \end{array} \right|$$

$$-2(R_1) + R_3 \rightarrow R_3$$

$$\begin{array}{cccc} -2 & 4 & 6 & 0 \\ 2 & 5 & 4 & 12 \\ \hline 0 & 9 & 10 & 12 \end{array}$$

MATRIX > MATH > E:* row+(-2,[A],1,3)

Verify output is correct

STO→[A]

$$\left| \begin{array}{cccc} 1 & -2 & -3 & 0 \\ 0 & 7 & 5 & 1 \\ 0 & 9 & 10 & 12 \end{array} \right|$$

$$1/7(R_2) \rightarrow R_2$$

MATRIX > MATH > F:* row(1/7,[A],2)

Verify output is correct

STO→[A]

$$\left| \begin{array}{cccc} 1 & -2 & -3 & 0 \\ 0 & 1 & .714 & .142 \end{array} \right|$$

$$\left| \begin{array}{cccc} 0 & 9 & 10 & 12 \end{array} \right|$$

to fractions

Change decimals MATH > ENTER > ENTER

Verify output is correct

STO→[A]

Updated matrix

Plan for this step

Calculator step

$$2(R_2) + R_1 \rightarrow R_1$$

$$\left| \begin{array}{cccc} 1 & -2 & -3 & 0 \\ 0 & 1 & 5/7 & 1/7 \\ 0 & 9 & 10 & 12 \end{array} \right|$$

$$\begin{array}{cccc} 0 & 2 & 10/7 & 2/7 \\ 1 & -2 & -3 & 0 \\ \hline 1 & 0 & -11/7 & 2/7 \end{array}$$

MATRIX > MATH > **F:*row+(2,[A],2,1)**

MATH > ENTER > ENTER

Verify output is correct. STO→[A]

$$-9(R_2) + R_3 \rightarrow R_3$$

$$\left| \begin{array}{cccc} 1 & 0 & -11/7 & 2/7 \\ 0 & 1 & 5/7 & 1/7 \\ 0 & 9 & 10 & 12 \end{array} \right|$$

$$\begin{array}{cccc} 0 & -9 & -45/7 & -9/7 \\ 0 & 9 & 10 & 12 \\ \hline 0 & 0 & 25/7 & 75/7 \end{array}$$

MATRIX > MATH > **F:*row+(-9,[A],2,3)**

MATH > ENTER > ENTER

Verify output is correct. STO→[A]

$$\left| \begin{array}{cccc} 1 & 0 & -11/7 & 2/7 \\ 0 & 1 & 5/7 & 1/7 \\ 0 & 0 & 25/7 & 75/7 \end{array} \right|$$

$$7/25(R_3) \rightarrow R_3$$

MATRIX > MATH > **E:*row(7/25,[A],3)**

MATH > ENTER > ENTER

Verify output is correct. STO→[A]

$$11/7(R_3) + R_1 \rightarrow R_1$$

$$\left| \begin{array}{cccc} 1 & 0 & -11/7 & 2/7 \\ 0 & 1 & 5/7 & 1/7 \\ 0 & 0 & 1 & 3 \end{array} \right|$$

$$\begin{array}{cccc} 0 & 0 & 11/7 & 33/7 \\ 1 & 0 & -11/7 & 2/7 \\ \hline 1 & 0 & 0 & 5 \end{array}$$

MATRIX > MATH > **F:*row+(11/7,[A],3,1)**

MATH > ENTER > ENTER

Verify output is correct. STO→[A]

$$-5/7(R_3) + R_2 \rightarrow R_2$$

$$\left| \begin{array}{cccc} 1 & 0 & 0 & 5 \\ 0 & 1 & 5/7 & 1/7 \\ 0 & 0 & 1 & 3 \end{array} \right|$$

$$\begin{array}{cccc} 0 & 0 & -5/7 & -15/7 \\ 0 & 1 & 5/7 & 1/7 \\ \hline 0 & 1 & 0 & -2 \end{array}$$

MATRIX > MATH > **F:*row+(-5/7,[A],3,2)**

MATH > ENTER > ENTER

Verify output is correct. STO→[A]

The matrix is now in reduced-row-echelon form. Rewrite the system of equations and simplify.

$$\left| \begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 3 \end{array} \right|$$

$$1x + 0y + 0z = 5 \quad \rightarrow \quad x = 5$$

$$0x + 1y + 0z = -2 \quad \rightarrow \quad y = -2$$

$$0x + 0y + 1z = 3 \quad \rightarrow \quad z = 3$$

Write answer as an ordered triplet. $(5, -2, 3)$