

## Solving Radical Equations

Radical equations are equations that contain radical expressions. The radical equations we are going to solve are mainly square root equations and cubic root equations.

Example #1: Solve  $\sqrt{x} = 8$

Solution:

The first thing we need to do to solve radical equations is to remove the radical ( $n$ th roots).

$$\sqrt{x} = 8$$

To remove the square root on the left side, we will need to square both sides of the equation.

$$(\sqrt{x})^2 = (8)^2$$

Simplify each side of the equation.

$$x = 64$$

Check the answer.  $x = 64$  is the solution.

Example #2: Solve  $\sqrt{2x - 5} = 3$

Solution:

This equation looks a little different than the previous one. The **radicand** (the expression under the radical sign) of the previous equation is  $x$ . The radicand of this equation is  $2x - 5$ . But, if the **radical term** is isolated, we can follow the same steps to solve the equation as mentioned above.

$$\sqrt{2x - 5} = 3$$

To remove the square root on the left side, we will need to square both sides of the equation.

$$(\sqrt{2x - 5})^2 = 3^2$$

Simplify each side of the equation.

$$2x - 5 = 9$$

Solve for  $x$ .

$$x = 7$$

Check your answer in original equation.

$$\sqrt{2(7) - 5} = 3$$

$$\sqrt{9} = 3 \checkmark$$

Example #3: Solve  $\sqrt{2x + 8} = x$

Solution:  $\sqrt{2x + 8} = x$  To remove the square root, square both sides.

$(\sqrt{2x + 8})^2 = (x)^2$  Simplify each side of the equation.  
 $2x + 8 = x^2$

$x^2 - 2x - 8 = 0$   
 $(x - 4)(x + 2) = 0$  To solve a quadratic, we need to set the equation equal to zero.  
 $x - 4 = 0$      $x + 2 = 0$   
 $x = 4$          $x = -2$

Then we can factor & solve for x.

$$\begin{aligned} \sqrt{2(4) + 8} &= 4 \\ \sqrt{16} &= 4 \\ 4 &= 4 \end{aligned}$$

We must check the solutions to see if they work. If they are a solution, we will get a true statement when we substitute them into the original equation.

$$\begin{aligned} \sqrt{2(-2) + 8} &= -2 \\ \sqrt{4} &= -2 \\ 2 &\neq -2 \end{aligned}$$

We can see that 4 is a solution, but  $-2$  is not.

**Exercises: Solve the following radical equations.**

1.  $\sqrt{3y - 1} = 5$     2.  $\sqrt[3]{x - 4} = -2$     3.  $\sqrt{x + 2} = x$

**Solutions:**

1.  $y = \frac{26}{3}$             2.  $x = -4$             3.  $x = 2$