Prime Factorization

Objective: Find the prime factorization of a natural number Important

Ideas:

1. Finding the prime factorization of a number means rewriting the number as a multiplication that uses only prime numbers as factors.

2. Prime numbers are numbers that have only two factors, the number 1 and the number itself. This means that they can only be divided by 1 and the number itself.

   The first 15 prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47 ….

3. Prime factorization is helpful for finding the Least Common Denominator when adding or subtracting fractions. It is also useful in multiplying, dividing and reducing fractions to lowest terms.

Finding the Prime Factorization

To find the prime factorization of a number follow these steps:

1. Start dividing the number you are factoring by the smallest prime number which will divide in evenly.

2. Continue to divide by that number as long as possible. When it will no longer work, go to the next highest prime number that will divide in evenly.

3. When the final quotient is a prime number, you are finished dividing.

4. If the number you are trying to divide by, multiplied by itself, is larger than the number you are dividing, the number you are trying to divide is a prime number.

5. Write the number as a product of its prime factors.

The following **divisibility tests** may be helpful to you.

1. If the number is an even number it can be divided by 2.

2. If the sum of the digits in a number is divisible by 3, then the number itself can be divided by 3.

3. If the number ends in 0 or 5, then the number can be divided by 5.

We will now work through several examples following each step in the process.

Example 1 Find the prime factorization of 144

We start dividing by the smallest prime number that will divide 144 evenly. As 144 is an even number we can divide by 2.

We keep dividing by 2 as long as we can.

$$\begin{array}{r} 9 \\ 2\overline{)18} \\ 2\overline{)36} \\ 2\overline{)72} \\ 2\overline{)144} \end{array}$$

2 will not divide evenly into 9, so we go to the next highest prime which will work. In this case that number is 3.

3 ← 3 is prime

$$\begin{array}{r} 3\overline{)9} \\ 2\overline{)18} \\ 2\overline{)36} \\ 2\overline{)72} \\ 2\overline{)144} \end{array}$$

The final quotient of 3 is prime so we have completed the division process. We now write 144 as a product of its prime factors.

$$144 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3$$

Example 2: Find the prime factorization of 300.

5 is prime → 5

3 will not work so try the next highest prime (5) → $5\overline{)25}$

2 will not work so try the next highest prime (3) → $3\overline{)75}$

150 is even so divide by 2 → $2\overline{)150}$

300 is even so divide by 2 → 2 300

$$300 = 2 \cdot 2 \cdot 3 \cdot 5 \cdot 5$$

Example 3: Find the prime factorization of 231

11 ← 11 is prime

77 is not divisible by 3 or 5, but it is divisible by 7 → $7\overline{)77}$

231 cannot be divided by 2 but it can be divided → 3 231 by 3. (Remember the divisibility check for 3; 2+3+1=6. 6 is divisible by 3 so 231 is divisible by 3)

$$231 = 3 \cdot 7 \cdot 11$$

Example 4: Find the prime factorization of 625

5 is prime → 5

25 is also divisible by 5 → $5\overline{)25}$

125 is also divisible by 5 → $5\overline{)125}$

625

cannot be divided by 2 or 3, but it can → 5 625 be divided by 5 (remember the divisibility check for 5; the number ends in a 5 so it is divisible by 5).

$$625 = 5 \cdot 5 \cdot 5 \cdot 5$$

Example 5: Find the prime factorization of 89

89 is not even so it cannot be divided by 2.

8+9 = 17; 17 cannot be divided by 3, so 89 cannot be divided by 3.

89 does not end in 0 or 5 so it cannot be divided by 5.

89 cannot be divided evenly by 7.

The next highest prime number is 11, but as $11 \cdot 11$ is greater than 89, we do not need to try 11.

89 is a prime number.

Practice Exercises

Find the prime factorization of each number.

1. 108

2. 500

3. 243

4. 605

5. 97

6. 576

7. 147

8. 512

9. 169

10. 2625

Answers to Practice Problems

1. $108 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3$

2. $500 = 2 \cdot 2 \cdot 5 \cdot 5 \cdot 5$

3. $243 = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$

4. $605 = 5 \cdot 11 \cdot 11$

5. 97 is prime

6. $576 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3$

7. $147 = 3 \cdot 7 \cdot 7$

8. $512 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

9. $169 = 13 \cdot 13$

10. $2625 = 3 \cdot 5 \cdot 5 \cdot 5 \cdot 7$