

For Solving Linear Applications

Many real-life applications can be approximated by a linear model. Information can come in a variety of ways to enable us to create a model. These examples show some of the ways that we can build our model. Our basic equations that we can use are:

$$y = mx + b$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$y - y_1 = m(x - x_1)$$

Example 1: Given both m and b.

1. A student buys a basic phone that has a base charge of \$16.44 per month, with each minute of calling time costing them \$.20. Develop a model that can help predict the cost each month. We need 2 things for the linear model: slope and y-intercept (base/fixed costs).

a. What we know:

- a. The model is $C(x) = mx + b$.
- b. The base cost is \$16.44, so $b = 16.44$.
- c. The slope or the part that changes is \$.20 per minute. The number of minutes of calls will be x . This gives $m = .20$.

Thus, our cost equation is $C(x) = .20x + 16.44$

- b. If they know that they only have \$66.44 this month for the phone bill, how many minutes can they use?

If $C(x) = 66.44$, we need to solve this for x :

$$66.44 = .20(x) + 16.44$$

$$66.44 = .20(x) + 16.44$$

$$50.00 = .20x$$

$$\frac{50.00}{.20} = \frac{.20x}{.20}$$

$$x = 250$$

Thus, the student can use their phone for 250 minutes this month and stay on budget!

Example 2: Given b and a point.

2. The population of a small town in the panhandle of Florida has been showing an incline in the years from 2002 to 2023. In 2002, the population was 15,200. In 2023, a survey shows that the population has grown to 16,460.

Write a linear equation modelling the population of this town, $P(t)$, as a function of t , the number of years since 2002.

a. What we know:

- a. The model is $P(t) = mt + b$.
- b. Given: (2002, 15, 200) and (2023, 16, 460).
- c. The base population in 2002 is 15, 200 so $b = 15, 200$.
- d. The slope or the part that changes is calculated by using the two points we were given.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{16,460 - 15,200}{2023 - 2002} = \frac{1260}{21} = 60$$

Now we know that the small town is gaining 60 people per year.

Thus, our population equation is $P(t) = 60t + 15,200$

- b. Use your equation to predict the population in the year 2040 to help understand the need for more teachers, police officers, etc.
 - a. We know the model: $P(t) = 60t + 15,200$
 - b. To calculate t, the time since 2002, we take the given year and subtract: $2040 - 2002 = 38$
 - c. Substitute this into our model.

$$P(38) = 60(38) + 15,200$$

= 17,480. This is the predicted population in the year 2040.

Example 3: Given 2 Random Points

3. A company was doing a study on their advertising budget and its affect on sales. They noted that when they spent \$40,000 on advertising, they sold 100,000 bags of candy. They also noted that when they spent \$60,000, they sold 200,000 bags of candy. They would like to develop a linear model that will give them the correlation between advertising budget and sales.
 - a. We can pair the given information as two ordered pairs; (\$advertising, sales). This give us two points to work with. (40,000, 100,000) and (60,000, 200,000).
 - b. The model is $C(x) = mx + b$. We need to determine the slope (m) and the y-intercept (b).
 - c. $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{200,000 - 100,000}{60,000 - 40,000} = \frac{100,000}{20,000} = 5$, which we can interpret as 5 bags sold per dollar spent in advertising.
 - d. We can then calculate a b(y-intercept) for this model. We can use: $y - y_1 = m(x - x_1)$
 $y - 100,000 = 5(x - 40,000)$; using the point (40,000, 100,000)
 $y - 100,000 = 5x - 200,000$; then solve this for y.
 $y = 5x - 300,000$
 So our linear model is $C(x) = 5x - 300,000$.
 - e. If the company wants to sell 350,000 bags of candy, how much should they spend in advertising?

$C(x) = 5x - 300,000$ is the model.

$$350,000 = 5x - 300,000; \text{ we want } C(x) = 350,000$$

$$x = 130,000.$$

To interpret, the company should spend \$130,000 on advertising to reach their target goal of selling 350,000 bags of candy.

You Try:

A company sells laptops. They are trying to develop a model that will show them how many laptops will be sold if they spend a certain amount on marketing. They have data that shows that if they spend \$150,000, they will sell 65,000 laptops. If they spend \$175,000, they will sell 77,500 laptops. Calculate the linear model that shows the correlation between marketing and estimated laptops to be sold. ANS: $M(x) = .5x - 10,000$.