

Tests for Convergence of Series

Test for Divergence If $\lim_{n \rightarrow \infty} a_n \neq 0$ or fails to exist, then $\sum_{n=1}^{\infty} a_n$ diverges.

Geometric Series

$\sum_{n=1}^{\infty} ar^{n-1}$ converges for $|r| < 1$, $s = \frac{a}{1-r}$

Diverges for $|r| \geq 1$

p Series

$\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges for $p > 1$

Diverges for $p \leq 1$

Ratio Test

If $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = L$

- i. The series is Absolutely convergent if $L < 1$ and therefore is convergent
 - ii. The series diverges if $L > 1$ or is infinite
 - iii. The test is inconclusive if $L = 1$
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Root Test

If $\lim_{n \rightarrow \infty} \sqrt[n]{|a_n|} = L$

Absolute Convergence

If $\sum_{n=1}^{\infty} |a_n|$ converges, then $\sum_{n=1}^{\infty} a_n$ converges absolutely and is convergent.

Integral Test

Suppose f is a continuous, positive, decreasing function on $[1, \infty)$ and let $a_n = f(n)$.

- i) If $\int_1^{\infty} f(x) dx$ is convergent, then $\sum_{n=1}^{\infty} a_n$ is convergent
- ii) If $\int_1^{\infty} f(x) dx$ is divergent, then $\sum_{n=1}^{\infty} a_n$ is divergent.

The Limit Comparison Test

Suppose $\sum a_n$ and $\sum b_n$ are series with positive terms.

If $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = c$

where c is a finite number and $c > 0$ then both series converge or both series diverge.

The Comparison Test

Suppose $\sum a_n$ and $\sum b_n$ are series with positive terms.

- i) If $\sum b_n$ is convergent and $a_n \leq b_n$ for all n , then $\sum a_n$ is also convergent.
 - i) If $\sum b_n$ is divergent and $a_n \geq b_n$ for all n , then $\sum a_n$ is also divergent.
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Alternating Series Test

If the alternating series $\sum_{n=1}^{\infty} (-1)^{n-1} b_n = b_1 - b_2 + b_3 - b_4 + \dots$ $b_n > 0$ satisfies

- i) $b_{n+1} \leq b_n \quad \forall n$
- ii) $\lim_{n \rightarrow \infty} b_n = 0$ then the series is convergent.