Using the Quotient Rule to find the Derivative

The Process for the Quotient Rule:

1. Given$H(x) = \frac{f(x)}{g(x)}$ then
2. Identify $f(x), f'(x), g(x), and\ g'(x)$
3. $H'(x) = \frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f\prime(x)-f(x)g\prime(x)}{[g(x)]^2}$; Quotient Rule
4. Substitute and simplify

Some other helpful rules:

$\frac{d}{dx}(c) = 0$ $\frac{d}{dx}[cf(x)] = c\,f'(x)$ $\frac{d}{dx}(x^n) = nx^{n-1}dx$ $\frac{d}{dx}(e^x) = e^x$

$\frac{d}{dx}\ln|x| = \frac{1}{x}dx$ $\frac{d}{dx}(a^x) = a^x\ln(a)$ $\frac{d}{dx}(\sin x) = \cos x\,dx$ $\frac{d}{dx}(\cos x) = -\sin x\,dx$

Algebraic Example of Quotient Rule:

$Given: H(x) = (x^2+1)/(x^3+6)^7$

$f(x) = x^2 + 1$ $g(x) = (x^3+6)^7$

$f'(x) = 2x$ $g'(x) = 7(x^3+6)^6(3x^2) = 21x^2(x^3+6)^6$; *using the chain rule*

$H'(x) = \frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f\prime(x)-f(x)g\prime(x)}{[g(x)]^2}$*; substitute the individual pieces*

$H'(x) = \frac{(x^3+6)^7(2x)-(x^2+1)21x^2(x^3+6)^6}{[(x^3+6)^7]^2}$*; then simplify*

$H'(x) = \frac{x(x^3+6)^6[(2)(x^3+6)-(x^2+1)21x]}{[(x^3+6)^7]^2}$*; factor out like terms before combining*

$H'(x) = \frac{x(x^3+6)^6[(2)(x^3+6)-(x^2+1)21x]}{[(x^3+6)^7]^2}$*;*

$H'(x) = \frac{x(x^3+6)^6[2x^3+12-21x^3-21x]}{(x^3+6)^{14}} \rightarrow \frac{x[12-19x^3-21x]}{(x^3+6)^8}$

Trig components Example:

$Given: H(x) = \frac{\ln(x)}{x^2}$

$f(x) = ln(x)$ $g(x) = x^2$

$f'(x) = \frac{1}{x}$ $g'(x) = 2x$

$H'(x) = \frac{x^2(\frac{1}{x})-\ln(x)(2x)}{[x]^2}$; simplify

$H'(x) = \frac{x-\ln(x)(2x)}{[x^2]^2} \rightarrow \frac{x(1-2\ln(x))}{[x]^4} \rightarrow \frac{1-2\ln(x)}{[x]^3}$

Quotient Rule/Radical Example:

$Given: H(x) = \frac{x^3+3}{\sqrt{x^2+7}}$; When you are working with radicals, the following exponent rules may come in handy: $\sqrt[b]{m^a} = m^{\frac{a}{b}}$ *and* $m^{-a} = \frac{1}{m^a}$

$f(x) = x^3 + 3$ $\quad$ $g(x) = (x^2+7)^{\frac{1}{2}}$

$f'(x) = 3x^2$ $\quad$ $g'(x) = \frac{1}{2}(x^2+7)^{-\frac{1}{2}}(2x) = x(x^2+7)^{-\frac{1}{2}} =$

$H'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$; so, we have:

$H'(x) = \frac{(x^2+7)^{\frac{1}{2}}(3x^2) - (x^3+3)(x(x^2+7)^{-\frac{1}{2}})}{[(x^2+7)^{\frac{1}{2}}]^2}$; then we begin simplifying and re-writing (if necessary)

$H'(x) = \frac{\sqrt{(x^2+7)}\,(3x^2) - (x^3+3)\left[\frac{x}{\sqrt{(x^2+7)}}\right]}{x^2+7}$; getting a common denomination

$H'(x) = \frac{\sqrt{(x^2+7)}\,(3x^2)\left[\frac{\sqrt{(x^2+7)}}{\sqrt{(x^2+7)}}\right] - (x^3+3)\left[\frac{x}{\sqrt{(x^2+7)}}\right]}{x^2+7}$; continuing to simplify

$H'(x) = \frac{\left[\frac{(3x^2)(x^2+7)}{\sqrt{(x^2+7)}}\right] + \left[\frac{-x^4-3x}{\sqrt{(x^2+7)}}\right]}{x^2+7}$; continuing to simplify

$H'(x) = \frac{\left[\frac{2x^4+21x^2-3x}{\sqrt{(x^2+7)}}\right]}{x^2+7} \rightarrow \left[\frac{2x^4+21x^2-3x}{\sqrt{(x^2+7)}}\right] \cdot \left[\frac{1}{x^2+7}\right] \rightarrow \frac{2x^4+21x^2-3x}{(x^2+7)^{\frac{3}{2}}}$

You try's:

1. $\mathrm{H(x)} = \frac{1}{\sqrt{25-y^2}}$
2. $\mathrm{H(x)} = \frac{sec^2 x}{1+x^2}$
3. $\mathrm{H(x)} = \frac{x-3}{x+7}$
4. $\mathrm{H(t)} = \frac{lnx}{(x+1)^4}$

Solutions:

1. $H'(x) = \frac{y}{(25-y^2)^{\frac{3}{2}}}$ **2.** $H'(x) = \frac{2sec^2(x)[tan(x) - x + tan(x)]}{(1+x^2)^4}$ **3.** $H'(x) = \frac{10}{(x+7)^2}$ **4.** $H'(x) = \frac{\frac{x+1}{x} - 4\,ln(x)}{(x+1)^5}$