Summary of Integration by Substitution

Steps to integration by substitution:

Example 1: Consider $\int (x^2+1)\, 2x\, dx$

1) Let u equal the expression inside the parenthesis.
   Solution: **u =** $\boldsymbol{x^2+1}$

2) Find du. Solution: **du = 2x dx**

3) Substitute. Solution: $\int \boldsymbol{(x^2+1)2x\, dx} = \int \mathbf{u\, du}$

   ↑ **u** ↑ **du**

4) Take the antiderivative of u. Solution: $\frac{u^2}{2} + c$

5) Substitute x^2+1 back in for u.

Final Solution: $\boldsymbol{\frac{(x^2+1)^2}{2} + c}$

Example 2: Consider $\int 3x\, e^{x^2} dx$

1) Let u equal the expression inside the exponent. Solution: **u =** $\boldsymbol{x^2}$

2) Find du. Solution: **du = 2x dx**

3) We need to be able to substitute something in for 3x dx. But du = 2x dx. So use algebra to get the right side of #2 to equal 3x dx.

$$\frac{3}{2}\, du = \frac{3}{2} * 2x\, dx \qquad \text{so} \qquad \boldsymbol{\frac{3}{2}\, du = 3x\, dx}$$

4) Substitute. Solution: $\int \mathbf{3x\,dx\,e^{x^2}} \quad = \quad \int \frac{3}{2}\,\mathbf{du\,e^{u}} \quad = \quad \frac{3}{2}\int \mathbf{du\,e^{u}}$

$\uparrow \quad \uparrow$

$\frac{3}{2}\mathbf{du}\ \ \mathbf{e^{u}}$

5) Take the antiderivative of e^u. Solution: $\frac{3}{2}\boldsymbol{e^{u}} + \boldsymbol{C}$

6) Substitute x^2 back in for u.

Final Solution: $\frac{3}{2}\boldsymbol{e^{x^2}} + \boldsymbol{c}$

Example 3: Consider $\int 4x \sin(x^2)\ dx$

1) Let u equal x^2. Solution: **u =** $\boldsymbol{x^2}$

2) Find du. Solution: **du = 2x dx**

3) We need to be able to substitute something in for 4x dx. But du = 2x dx. So use algebra to get the right side of #2 to equal 4x dx.

$$2\ \mathrm{du} = 2 * 2\mathrm{x}\ \mathrm{dx} \qquad \text{so} \qquad \mathbf{2\ du = 4x\ dx}$$

4) Substitute.

Solution: $\int \mathbf{4x\ dx\ sin(x^2)} \quad = \quad \int \mathbf{sin(u) 2du} \quad = \quad = \mathbf{2}\int \mathbf{du\ sin\ (u)}$

$\uparrow \qquad \uparrow$

2 du sin(u)

5) Take the antiderivative of sin(u). Solution: $\mathbf{2}(-\boldsymbol{cos}(\boldsymbol{u})) = -\mathbf{2}\boldsymbol{cos}(\boldsymbol{u}) + \boldsymbol{C}$

6) Substitute x^2 back in for u.

Final Solution: $-\mathbf{2cos}\ (\boldsymbol{x^2}) + \boldsymbol{C}$

Example 4: Consider $\int \frac{2}{x} \ln(x^2)\, dx$

1) Let u equal $\ln(x^2)$. Solution: **u =** $\mathbf{\ln(x^2)}$

2) Find du. Solution: **du =** $\mathbf{\frac{1}{x^2} 2x\ \ dx}$ **=** $\mathbf{\frac{2}{x}\ dx}$

3) We need to be able to substitute something in for $\frac{2}{x}$ dx. But du = $\frac{2}{x}$ dx.

$$\mathbf{du = \frac{2}{x}\ dx}$$

4) Substitute.

Solution: $\int \mathbf{\frac{2}{x}\ dx\ \ln(x^2)} \quad = \quad \int \mathbf{u\ \ du}$

↑ ↑

du **u**

5) Take the antiderivative of u.

Solution: $\mathbf{\frac{u^2}{2} + C}$

6) Substitute $\ln(x^2)$ back in for u.

Final Solution: $\mathbf{\frac{(\ln(x^2))^2}{2} + C}$

Practice Problems:

Find the Indefinite Integral using substitution:

1) $\int 5x(1+\ x^2)^3\ dx$

2) $\int -x^3(2-\ x^4)^2\ dx$

3) $\int 3x^2 \cos(x^3)\ dx$

4) $\int -6x\, e^{2x^2}\ dx$

5) $\int \frac{3}{x} \ln(x^3)\ dx$

Solutions:

1) $\frac{5(1+x^2)^4}{8} + C$

2) $\frac{(2-x^4)^3}{12} + C$

3) $\sin(x^3) + C$

4) $\frac{-3}{2} e^{2x^2} + C$

5) $\frac{(\ln(x^3))^2}{2} + c$