Using the Chain Rule to find the Derivative

The Process:

1. Given: $F(x) = f(x)^\circ g(x)$ or $F(x) = f(g(x))$
2. Identify $f(x), f'(x), g(x), and\ g'(x)$
3. $F'(x) = f'(g(x)) \cdot g'(x)$
4. Substitute and simplify

Some other helpful rules:

$\frac{d}{dx}(c) = 0$ $\frac{d}{dx}[cf(x)] = c\,f'(x)$ $\frac{d}{dx}(x^n) = nx^{n-1}dx$ $\frac{d}{dx}(e^x) = e^x$

$\frac{d}{dx}\ln|x| = \frac{1}{x}dx$ $\frac{d}{dx}(a^x) = a^x\ln(a)$ $\frac{d}{dx}(\sin x) = \cos x\,dx$ $\frac{d}{dx}(\cos x) = -\sin x\,dx$

<u>e^x Example:</u>

$Given: H(x) = e^{3x}$

$f(g(x)) = e^{g(x)}$ *g(x) = 3x*

$f'(g(x)) = e^{g(x)}$, by definition. $g'(x) = 3$

$H'(x) = f'(g(x)) \cdot g'(x)$ so, we have:

$H'(x) = e^{g(x)} \cdot 3$; then substitute g(x) and g'(x)

$H'(x) = e^{3x} \cdot 3$ or this can be written as $3e^{3x}$.

<u>Trig with Exponent Example:</u>

$Given: H(x) = sin(x^2)$

$f(g(x)) = \sin(g(x))$ $g(x) = x^2$

$f'(g(x)) = \cos(g(x))$; *by definition.* $g'(x) = 2x^1$, *using the power rule.*

$H'(x) = f'(g(x)) \cdot g'(x)$ so, we have:

$H'(x) = cos(g(x)) \cdot 2x$

$H'(x) = cos(x^2) \cdot 2x$ or this can be written as $2xcos(x^2)$

<u>Natural Log Example:</u>

$Given: H(x) = ln(x^2 + 3x)$

$f(g(x)) = \ln(g(x))$ *g(x)* $= x^2 + 3x$

$f'(g(x)) = \frac{1}{g(x)}$; by definition. $g'(x) = 2x + 3$

$H'(x) = f'(g(x)) \cdot g'(x)$ so, we have:

$H'(x) = \frac{1}{g(x)} \cdot (2x + 3)$; then substitute g(x) and g'(x)

$H'(x) = \frac{1}{x^2+3x} \cdot (2x + 3)$ or this can be written as $\frac{2x+3}{x^2+3x}$

Combination Power and Chain Rule Example

$Given: H(x) = (2 - x^2)^4$

$f\big(g(x)\big) = g(x)^4$ $\qquad$ $g(x) = 2 - x^2$

$f'(g(x)) = 4(g(x)^3$; using the power rule $\qquad$ $g'(x) = -2x$

$H'(x) = f'(g(x)) \cdot g'(x)$ so, we have:

$H'(x) = (4g(x)^3)(-2x);$ then substitute g(x) and g'(x)

$H'(x) = (4(2 - x^2)^3)\,(-2x)$; then we need to simplify

$H'(x) = (4)(\text{-}2x)(2 - x^2)^3 = -8x(2 - x^2)^3$

Radical exponent Example

$Given: H(x) = \dfrac{1}{\sqrt{2 - x^4}}$

We will also need to use two algebra/exponent rules: $m^{-a} = \frac{1}{m^a}$ $\quad and \quad$ $\sqrt[b]{m^a} = m^{\frac{a}{b}}$.

$f(g(x)) = \frac{1}{\sqrt{g(x)}} = g(x)^{-\frac{1}{2}}$ $\qquad$ $g(x) = 2 - x^4$

$f'(g(x)) = \frac{-1}{2}\big(g(x)\big)^{\frac{-3}{2}}$; using the power rule $\qquad$ $g'(x) = 8x^3$

$H'(x) = f'(g(x)) \cdot g'(x)$ so, we have:

$H'(x) = \frac{-1}{2}\big(g(x)\big)^{\frac{-3}{2}} \cdot (8x^3)$; then substitute g(x) and g'(x)

$H'(x) = \frac{-1}{2}(2 - x^4)^{\frac{-3}{2}} \cdot (8x^3)$; then we need to simplify

$H'(x) = \frac{-1}{2} \cdot (8x^3)(2 - x^4)^{\frac{-3}{2}} = \dfrac{-4x^3}{(2 - x^4)^{\frac{3}{2}}}$

You try's:

1. $\text{H(x)} = sin(x^2 + 6x + 3)$
2. $\text{H(x)} = e^{6x^2 - 4x}$
3. $\text{H(x)} = \ln(x^2 + 3)$
4. $H(t) = 2^{t^3}$

Solutions:

1. $H'(x) = (2x + 6)cos\,(x^2 + 6x + 3)$ 2. $H'(x) = e^{6x^2-4}(12x - 4)$
3. $H'(x) = \frac{2x}{x^2+3}$ 4. $H'(t) = 2^{t^3}(3t^2)ln\,(2)$