Coin and Stamp Problems

Coin and stamp problems involve different types of coins, or stamps with different values. We will use a table with a line for each kind of coin or stamp. Above the table, we will use an equation which shows the following relationship:

$$\left(\begin{array}{c}\text{The number} \\ \text{of} \\ \text{coins or stamps}\end{array}\right) \times \left(\begin{array}{c}\text{The value} \\ \text{of each} \\ \text{coin or stamp}\end{array}\right) = \left(\begin{array}{c}\text{Total Value of} \\ \text{the coins or} \\ \text{stamps}\end{array}\right)$$

EXAMPLE: A box contains $1.30 in dimes and nickels. How many dimes and how many nickels are there if there are 20 coins altogether?

The first step is to set up the table.

	number $\times$	value	$=$ total value
dimes			
nickels			
total			

Determine how the numbers of each kind of coin are related. **REMEMBER** that you can have only one variable.

Do we know the number of dimes? No. Do we know the number of nickels? No.
What **do** we know about the number of coins? We know they **total** 20 coins.

We will let $x =$ the number of dimes.
We will let $20 - x =$ the number of nickels.

	number $\times$	value	$=$ total value
dimes	x	10	$10x$
nickels	$20 - x$	5	$5(20 - x)$
total	20		130

NOTE that the value of each type of coin is in cents. Our equation will be:

$$\left(\begin{array}{c}\text{The total value} \\ \text{of dimes}\end{array}\right) + \left(\begin{array}{c}\text{The total value} \\ \text{of nickels}\end{array}\right) = \text{Total Value}$$

(See last column of the table to find expressions for the total value of each kind of coin.)

As we represented the value of each coin in cents we will have to change $1.30 to 130 cents. You must be consistent with this! Now there are no decimals.

$$10x+5(20-x)\ 130=$$

NOW SOLVE:

$$10x+5(20-x)\ 130=$$
$$10x+100\ 5-\ =x\ 130$$
$$5x+100\ 130=\ 5x+100\ 100$$
$$130\ 100-\quad =\quad -$$
$$5x=\ 30$$
$$\frac{5x}{5}=\frac{30}{5}\ x=6$$

The number of dimes $= 6$
The number of nickels $= 20 - 6 = 14$

CHECK: $6(10) = 60$ cents
$14(5) = \underline{70}$ cents $\quad + 130$
cents

EXAMPLE: A collection of stamps has some 5¢ stamps, some 8¢ stamps and 25¢ stamps. The number of 5¢ stamps is three times the number of 25¢ stamps. The number of 8¢ is five more than the number of 25¢ stamps. If the value of all the stamps is $3.76 how many of each kind are there?

Let us first determine how the numbers of each type of stamp are related. We can only use one variable.

Number of 5¢ $= 3$ times the number of 25¢
Number of 8¢ $= 5$ more than the number of 25¢
Number of 25¢ $= ?$

NOTE that the number of 5¢ stamps and the number of 8¢ stamps are described **<u>using</u>** the number of 25¢ stamps.

Let the number of 25¢ stamps $= x$ then
the number of 5¢ stamps $= 3x$ and the
number of 8¢ stamps $= x + 5$

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	number	×	value	=	total value
5¢ stamps	$3x$		5		$5(3x)$
8¢ stamps	$x + 5$		8		$8(x + 5)$
25¢ stamps	x		25		$25x$
total					376

The value of **all** the stamps = \$3.76 which is 376¢

Our equation will be:

$$5(3) 8(x + x+5) 25+ x= 376$$

Don't forget to change \$3.76 to 376 cents.

NOW SOLVE:

$$5(3) 8(x + x+5) 25+ x= 376$$
$$15x+ + +8x\ 40\ 25x= 376$$
$$48x+ =40\ 376$$
$$48x+ - =40\ 40\ 376\ 40-$$
$$48x= 336$$
$$\frac{48x}{48} = \frac{336}{48}$$
$$x= 7$$

There are 7 of the 25¢ stamps.
There are 3(7) or 21 of the 5¢ stamps
There are 7 + 5 or 12 of the 8¢ stamps